



Optimal regimes of heavy oil transportation through a heated pipeline

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ABSTRACT

Heavy oil with a high pour point and high viscosity is transported in a heated oil pipeline with many oil pumping and heating stations. Electricity and fuel costs account for a large share of total pumping cost. Minimizing the total operating cost of pumps and heating furnaces used to pump oil requires optimization of their hydrodynamic parameters, and, above all, the distribution of pressure and temperature along the pipeline. The application of graph theory and dynamic programming to determine the problem of the global optimum of energy consumption is given in a heated oil pipeline with many oil pumping and heating stations. The problem of mass and heat transport was solved numerically. The results of the optimization problem were applied to a digital section of a heated trunk interstate oil pipeline 450 km long. The results of thermohydraulic calculations were consistent with the actual data of the Supervisory Control and Data Acquisition (SCADA) system. Energy saving is 38.9 % in the optimal regime.

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1. Introduction

To transport a high-viscosity and high-pour-point oil through a pipeline, the hot pumping method is applied, which consists in heating oil with fired furnaces at heating stations and pumping liquid oil by pump units located at pumping stations. Several examples of such pipelines are known in the world as the Uzen–Atyrau–Samara trunk interstate pipeline of 1380 km long, which transports heavy oil from the oil field Uzen through the territory of Kazakhstan and Russia [1]. Another example is the Usa–Ukhta–Yaroslavl trunk intrastate pipeline in Russia of 1130 km long, which pumps heavy oil from the oil field Usen [2]. The third example is the Rizhao–Yizheng intrastate crude oil pipeline in China of 375 km long [3]. Several new projects of such heated pipelines are in development in China and India [4]. The minimization of costs related to heating and pumping for long distances is, obviously, of high practical importance [5].

The solution to the problem of optimization for cold transport is known [6], it is based on optimizing the pressure distribution along the pipe. In the case of hot transport, the energy consumed by heating furnaces is much higher in cost than the electricity required for pumping units [7]. Consequently, optimizing the temperature distribution becomes the main challenge. In reality, we have to optimize both temperature and pressure fields simultaneously since they are coupled (the heat is transported by moving oil).

Such a system really has a nontrivial optimal regime, which is explained as follows. If the oil temperature is too high, the heating costs become excessive. Conversely, if the oil temperature is too low, then heating costs fall, but pump power costs increase, as it becomes more difficult to pump viscous liquid [5].

Jefferson [8] posed the problem of choosing the optimal oil flow mode in an isothermal pipeline. He considered an oil pipeline equipped with pumps of various pressure characteristics. The optimal mode corresponds to such a pressure distribution between pump stations at which the electricity cost consumed by all pumps per unit time is minimal. The optimization procedure should take into account some technological constraints imposed on pressure value ahead of each pump station and after each control valve. To solve the optimization problem Jefferson [8] used of dynamic programming. As the result, a single combination of pump connections can be selected from all possible combinations that enables to consume minimum power at a given production rate and a given capacity of each station.

Gopal [9] analyzed the optimal choice of pump combination and discharge pressure using dynamic programming. Jokic and Zavargo [10] tried to optimize pipeline diameters to reduce the costs using non-linear programming.

Shepetkov [11] obtained the solution of the optimization problem by the linear programming method. Tumansky [12] considered the issue of pumping optimization in a pipeline with several pumping stations with variable frequency drive. For this task, due to nonlinear conditions, he used dynamic programming instead of linear programming.

Hydraulic calculations of an isothermal pipeline are performed in [13–16]. Several papers studied the impact of various factors

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on the optimal solution: energy consumption strategy [13,14], pipeline operating scheme [15], pumping parameters [16].

The problem of energy consumption in a heated pipeline was considered in [17–24]. The results depend on pumping parameters and oil properties [17–19], as well as on pipelines operation [20–22]. A two-level hierarchical model was developed in [23] and the HOPOPT software pack in [24].

The optimization problem for heated oil pumping was analyzed by Yablonsky et al. [25]. The optimal regime corresponds to the case when the total cost of pumping and heating per unit length is identical at the beginning and at the end of the pipeline.

Some solutions to the optimization problem in heated pipelines were also given in [26,27].

In [27], the optimization model of a heated oil pipeline taken into account the constraints on pump characteristics, oil temperature and pressure at outlet and inlet of pumps and heating stations. The optimization was carried out by the combination of linear approximation and simplex method. A practical application showed significant energy saving in the optimal mode.

A non-stationary model of heat and mass transport was used to optimize a heated oil pipeline in [28]. The example of the Rizhao–Yizheng pipeline was calculated. The relative error was 4.04% compared to the real time data of SCADA, which is much less than the error given by the steady-state model (22.67%). The optimized operation scheme can save 17.59% of energy cost for the oil rate of 2640 m³/h.

Online optimization based on a neural network was suggested in [29]. Instead of solving transport equations, the neural network was trained and built by using pipeline history data obtained from SCADA. The model was applied to the Rizhao–Yizheng pipeline. The maximum reduction of the total energy cost is 10.75% with a significant saving effect.

The optimization results were obtained in [25–29], but the costs of technological equipment pumping oil were not taken into account.

The simplest method to optimize pump operation is searching all pump combinations. The time of such calculation is proportional to 2 to the power equal to the number of all pumps n , $t = 2^n$. This approach, in the case of a large number of pumps, makes the calculation resource-intensive. For example, a single calculation of a pipeline section with 6 pumping stations with 4 pumps in each requires about an hour and a half, and also a parallel implementation is proposed. Where there is a VFD, the pump can produce a whole range of head values. In this case, the enumeration becomes so complicated that it makes the calculation completely impossible.

Some authors [30,31] suggested approaches based on genetic algorithms. However, as you know, the genetic algorithm has several disadvantages. For example, the fitness function in this algorithm is highly time-consuming for real high-dimensional problems. The exact condition for stopping the algorithm is unclear. Genetic algorithms tend to converge towards a local optimum, or even contentious points, instead of a global optimum.

In optimization theory, the Lagrange method is widely used for mathematical nonlinear programming problems. In our problem, the constraints imposed on the variables are not equations, but inequalities; therefore, to modify the method, the Karush–Kuhn–Tucker conditions must be satisfied. Moreover, in our case, the constraint functions are not continuously differentiable, which contradicts the mandatory conditions of the Lagrange method.

We have proposed a completely different approach based on the ideas of graph theory [32–35] and dynamic programming. It has been noticed that the problem of finding a power-saving regime can be broken into many overlapping sub-problems, with finding the optimal substructure. Using the problem solution for n pumps, you can effectively find solutions for $n + 1$ pumps. A

graph of the operation states of the pumping units was formed. Each node of the graph contains data on the number of used pumps and its parameters, pressure drop in a pressure regulator. These nodes were connected to each other based on the head-flow characteristic and rotational speeds of the pump rotors. The transition of the solution of the subproblem to the solution of the general problem was found and its correctness was proved.

In the present paper, we suggest an optimization methodology using graph theory and dynamic programming and a model of mass and heat transfer in a heated pipeline.

2. Setting the optimization problem

2.1. Structural diagram of a pipeline

The problem considers the technological sections of the Kassymov–Bolshoy Chagan oil pipeline (Fig. 1). On this oil pipeline there is only 1 starting point of the flow beginning (Kassymov station) and only 1 end point at the flow end (Bolshoy Chagan station). The oil flows through pipeline strictly in one direction from the initial station to the final station. There are intermediate stations along the pipeline: Karmanovo, Inder, Antonovo, Sakarniy, and Baranovka.

The stations of the heated pipeline have heating furnaces, centrifugal pumps and pressure regulators (Fig. 1). At the stations, pumps form a group of pumps, and heating furnaces form a group of heating furnaces. In the calculations operating pumps and heating furnaces can be selected from these groups. The placement of the heating furnace is possible before or after the pumps. The location of the heating furnace before the pump can lead to a change in the condition of cavitation-free operation of the pump [5].

2.2. Optimization criterion

The cost of oil transport consists of two components: the cost of oil pumping and that of oil heating. A pump consumes electricity, while a heating furnace consumes gas fuel. Consequently, the total cost is the cost of consumed electricity by pumps and fuel by heaters. If N is the power consumption by a pump and z^e is the cost of electricity, then the cost of pumping is $z^e N$. If z^f is the cost of fuel, and q^f is the rate of fuel consumption by a heater, then the cost of heating is $z^f q^f$. For n stations, the optimal mode corresponds to the minimum of the total cost:

$$\sum_{i=1}^n \left(z_i^e \sum_{j=1}^{m_i^p} \delta_{ij}^p N_{ij}(k_{ij}) + z_i^f \sum_{k=1}^{m_i^h} \delta_{ik}^h q_{ik}^f \right) \Rightarrow \min \quad (2.1)$$

where superscripts p and h mean *pump* and *heater* respectively; index i means the station ordering number; m_i^p is the number of pumps at the i th station; and m_i^h is the number of heaters at the i th station; z_i^e is the cost of electricity (\$/(kW h)) and z_i^f is the cost of fuel (\$/kg), respectively; q_{ik}^f is the rate of fuel consumption by the k th heater at the i th station (kg/h); $N_{ij}(k_{ij})$ is the power consumption of the j th pump at the i th station (kW); k_{ij} is the ratio of the rotor speed of this pump to its nominal rotor speed.

Some pumps can run or be temporarily stopped depending on the pressure field along the pipeline, and similarly, several heaters can be stopped or run depending on the temperature field. This is why parameters δ_{ij}^p and δ_{ik}^h have been introduced. They are equal to 1 if the pump or heater is in operation, and 0 otherwise.

The first term in Eq. (2.1) determines the energy cost consumed by pumps to create hydraulic head (pressure), while the second term is the fuel cost of heating high-pour-point and high-viscosity oil at each station.

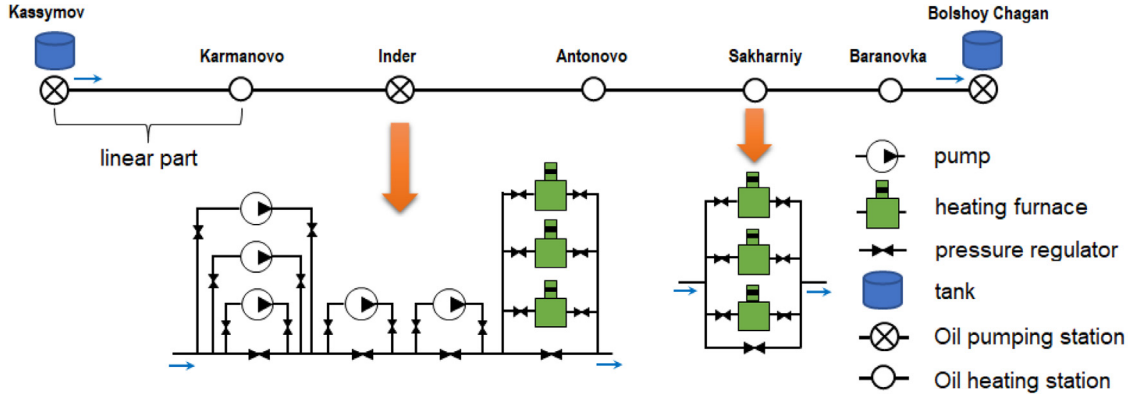


Fig. 1. Structural diagram of a pipeline: tanks, linear part, stations, heating furnaces, pumps, pressure regulators.

Functions $N_{ij}(k_{ij})$ (the power consumption) and q_{ij}^f (the rate of fuel consumption) depend on the oil flow rate q via pressure and temperature and on the structural combination of pumping units and heating furnaces. Therefore, the minimization of the functional (2.1) provides the optimal value of q and the optimal structural architecture of the pumping–heating system.

The criterion (2.1) should be supplemented by several technological conditions on pressure, temperature and other parameters.

2.3. Restriction on the pressure

The pressure in the overall system can be neither too high nor too low. More precisely, the restriction on the pressure is the following:

- the pressure at the inlet of the k th pumping station should be higher than a minimum admissible value $\Delta P_k^{\text{in},\text{min}}$:

$$P_k^{\text{in}} \geq \Delta P_k^{\text{in},\text{min}} \quad (2.2)$$

- the pressure at the outlet of the k th pumping station should be lower than a maximum admissible value $\Delta P_k^{\text{out},\text{max}}$, which concerns both the pressure behind and ahead of the pressure regulator (see. Fig. 1):

$$P_k^{\text{out}1} \leq P_k^{\text{out},\text{max}}, \quad P_k^{\text{out}2} \leq P_k^{\text{out},\text{max}} \quad (2.3)$$

■ to avoid the cavitation, the pressure must be higher than a critical cavitation-free value, for each l th pumping group of k th station:

$$P_{kl}^{\text{in},\text{gr}} \geq P_{kl}^{\text{min},\text{in},\text{gr}} \quad (2.4)$$

2.4. Restriction on the temperature

There is a certain upper temperature limit $T_{\text{oil}}^{\text{max}}$, below which the pumped oil does not decompose into separate fractions and does not evaporate. On the other hand, oil must be hotter than the pour point ($T_{\text{oil}}^{\text{min}}$). Thus:

$$T_{\text{oil}}^{\text{min}} < T(x, t) \leq T_{\text{oil}}^{\text{max}} \quad (2.5)$$

where $T(x, t)$ is the oil temperature in the pipeline over time t and length x .

2.5. Other restrictions

- The safety restriction on the furnace operation requires that:

$$q_{\text{min},ij}^f \leq q_{ij}^f \leq q_{\text{max},ij}^f \quad (2.6)$$

- Restriction on pump operating frequency:

Parameter k_{ij} cannot be more than one since the frequency converter is only able to reduce the rotor speed, so that:

$$k_{ij}^{\text{min}} \leq k_{ij} \leq 1 \quad (2.7)$$

where k_{ij}^{min} is the minimum allowable value that defines the lower limit for a variable frequency drive (VFD) for any pump. The VFD is an electrotechnical device that enables to change the frequency of a pump and its productivity below the nominal value. If the pump is not running in VFD mode (at a constant nominal frequency) then $k_{ij} = 1$.

The number of simultaneously operating pumps in VFD mode is limited by the number of drives p in the station. The number of pumps adapted to a VFD cannot be more than the number of drives. Therefore, for each i th station, it is necessary to set a limit on the number of simultaneously operating pumps in VFD mode:

$$\sum_{j=1}^{m_i} c_{ij} \lceil 1 - k_{ij} \rceil \leq p_i \quad (2.8)$$

where the operator $\lceil \cdot \rceil$ means the operation of rounding down, c_{ij} is an integer variable that takes on the value 1 if the pump is in operation, and 0 otherwise.

The restriction of the power consumption allows avoiding the overload of electric motors, which means the excess of power consumption compared to the rated power of the electric motor N

$$N \leq N^{\text{NOM}} \quad (2.9)$$

3. Model of mass and heat transfer

The system of restrictions is formulated in terms of pressure and temperatures, while the optimizing parameter is the flow rate. The relation between them can be found from the mass and heat transport equations.

3.1. Mass and heat transport in the linear part of the pipeline

The length of pipeline is about thousand kilometers, while the diameter is of 1 m. Therefore, a one-dimensional model can be applied.

The flow continuity equation is:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} = 0 \quad (3.1)$$

The momentum balance for turbulent flow is:

$$\rho \frac{\partial u}{\partial t} + \frac{\partial P}{\partial x} = -\zeta(\text{Re}, \varepsilon) \frac{\rho u |u|}{2D_1} - \rho g \sin \beta(x) \quad (3.2)$$

which is an equation like Navier–Stokes with the inertial and friction resistance terms taken in the empirical form quadratic with respect to the flow velocity.

The heat transfer is described by the equation:

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} = -\frac{4k}{\rho c_p D_1} (T - T_w) + \frac{\zeta u^3}{2c_p D_1} \quad (3.3)$$

where p , u and T are the pressure, the flow velocity, and the oil temperature ($^{\circ}\text{C}$); c_p is the heat capacity; $\zeta = \zeta(\text{Re}, \varepsilon)$ is the coefficient of hydraulic resistance; $\text{Re} = \rho u D / \mu$ is the Reynolds number; k is the parameter of heat transfer with soil; ε is the parameter of roughness of tube walls; D_1 is the pipe diameter; T_w is the soil temperature; $\beta = \beta(x)$ is the slop angle of the pipeline.

This equation describes the convective heat transport (on the left), the heat exchange with soil (the first term on the right), and the dissipation of kinetic energy into heat (the last term on the right).

3.2. Reduced model for incompressible oil

A single assumption enables us to simplify the model:

We assume that oil can be considered as an incompressible fluid.

Indeed, the experimental data on the oil density are given by the standard formula: $\rho(T) = \rho_0 [1 + \xi (20 - T)]$, where $\xi = 0.000738 (1/^{\circ}\text{C})$ is the oil volume expansion coefficient. In the range of oil temperature variation in the industrial pipeline from 20 to 50 $^{\circ}\text{C}$, the ratio ρ/ρ_0 varies between 1 and 0.98, therefore the oil density remains practically constant.

Corollaries:

■ The flow velocity and the flow rate are constant along x and over time:

$$u = \text{const}, \quad Q = \text{const} \quad (3.4)$$

which results from (3.1) and from the boundary condition for u : we accept the condition of an inlet velocity invariable in time, $u(0, x) = u_0 = \text{const}$.

■ The pressure field is quasi-stationary, which results from (3.1) and (3.2).

In this case, the system (3.1)–(3.3) is simplified and takes the form:

$$\frac{\partial P}{\partial x} = -\zeta \frac{\rho_0 u^2}{2D_1} - \rho_0 g \sin \beta(x) \quad (3.5)$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} = -\frac{4k}{\rho_0 c_p D_1} (T - T_w) + \frac{\zeta u^3}{2c_p D_1} \quad (3.6)$$

The initial and boundary conditions are:

$$T(x, 0) = T_w(x), \quad 0 \leq x \leq L \quad (3.7)$$

$$T(0, t) = T_0, \quad P(0, t) = P_0, \quad 0 \leq t \leq T_s \quad (3.8)$$

Model (3.5), (3.6) describes the flow of an incompressible fluid with quasi-stationary field of pressure, but with non-stationary temperature field. The unknown variables are: $P(x, t)$ and $T(x, t)$. The flow velocity u (and the flow rate Q) is a given constant value.

3.3. Closure relations

Experimental data on the dependence of oil viscosity on the temperature are described by the empirical formula:

$$\mu(T) = 0.3585 \cdot \exp(-0, 1792 \cdot T), \quad [\text{Pa s}] \quad (3.9)$$

The coefficient of hydraulic resistance $\zeta(\text{Re}, \varepsilon)$ is found by Altshul's formula [36]:

$$\zeta(\text{Re}, \varepsilon) = 0.11 \cdot \left(\frac{68}{\text{Re}} + \varepsilon \right)^{0.25} \quad (3.10)$$

Altshul's formula is in correlation with the Colebrook–White formula [37].

In a heated pipeline, the heat transfer coefficient k largely depends on the external coefficient of heat transfer with the soil α_2 . This coefficient is determined by the formula of Forchheimer–Greber [38]:

$$k = \alpha_2 \frac{D_2}{D}, \quad \alpha_2 = \frac{2\lambda_w}{D_2 \ln \left[\frac{2H}{D_2} + \sqrt{\left(\frac{2H}{D_2} \right)^2 - 1} \right]} \quad (3.11)$$

where D_2 is the outer diameter of the pipeline; λ_w is the soil thermal conductivity; H is the depth of the pipeline in the soil.

Eq. (3.11) is the truncated part of the most general equation $1/kD = 1/\alpha_1 D + 1/\alpha_2 D_2 + \sum_{i=1}^n (1/2\lambda_i) \ln(D_{i+1}/D_i)$, in which D_i and λ_i is the outer diameters and the thermal conductivity of the i th insulation layer.

The soil thermal conductivity coefficient λ_w in the formula (3.11) depends on the type and moisture of soil [5]. The identification of λ_w along the pipeline length was carried out by solving the inverse problem of the unsteady heat transfer Eq. (3.3). In the calculations, the distributions of oil temperature $T(x, t)$ were obtained, which were compared with the actual SCADA data. As a result of solving the inverse problem, the values of the soil thermal conductivity coefficient λ_w were found.

Experimental data on the heat capacity of heavy oil are described by the Kregó's formula:

$$c_p(T) = (53357 + 107.2 \cdot T) / \sqrt{\rho_0} \quad [\text{J}/(\text{kg } ^{\circ}\text{C})] \quad (3.12)$$

3.4. Pressure and temperature jumps in structural elements

Along with the pressure and temperature in the linear part of the pipeline, we need to calculate the jumps in various devices of the system:

- the increase in pressure created by the j th pump group of the i th station, $\Delta P_{ij}^{\text{gr}}$;
- the pressure loss after the pressure regulator, ΔP_i^{PR} ;
- the pressure loss at the i th oil heating station (OHS), ΔP_i^{OHS} .
- the back pressure value at the inlet of the end station, ΔP^{bp} ;

All these values are calculated using empirical formulae calibrated for each specific device and attached to their technical data sheets.

The temperature jumps in the heating stations are calculated in the following way:

■ the increase in the oil temperature through a heating furnace, ΔT^{h} :

$$\Delta T^{\text{h}} = C^{\text{h}} Q^{\text{h}} q^{\text{h}} \quad (3.13)$$

where Q^{h} is the flow rate through the furnace, q^{h} is fuel supply rate; C^{h} is a constant that depends on the furnace construction; it is proportional to the efficiency coefficient of the heating furnace.

The certain temperature limit of pumped oil exists, up to which it can be safely heated in the furnace until it does not decompose into separate fractions or does not evaporate. Let us designate this value $T_{\text{oil}}^{\text{max}}$. This condition is taken into account by limiting for each j th furnace at the i th station:

$$T_i + \Delta T_{ij}^{\text{h}} = T_i + C_{ij}^{\text{h}} Q_{ij}^{\text{h}} q_{ij}^{\text{f}} \leq T_{\text{oil}}^{\text{max}} \quad (3.14)$$

The increase in the oil temperature through an overall heating station, which includes m^h furnaces, is calculated as the average over all furnaces of the station:

$$\Delta T^{\text{OHS}} = \sum_{i=1}^{m^h} g_i^h \frac{Q_i^h}{Q} \Delta T_i^h \quad (3.15)$$

where the coefficients g_i^h take into account the operating mode of the furnaces.

Formula (3.14) takes into account the fact that a part of oil can pass outside furnaces and remains cold. This cold oil contributes into Eq. (3.15) by the fact that $\sum_{i=1}^{m^h} Q_i^h < Q$.

3.5. Numerical solution to the mass and heat transport problem

Problem (3.5)–(3.8) was solved numerically by the finite difference method [39,40]. The numerical code was implemented in the SmartTran software [41]. A four-core Intel (R) Xeon (R) CPU E3-1250 V5 @ 3.40 GHz was used for optimization calculations.

4. Solving the optimization problem

The problem was solved using graph theory and dynamic programming. The search for an energy-saving mode can be broken down into many overlapping sub-problems, with the search for the optimal substructure.

The object of each subtask is the consumed energy cost dependence function $S(P, T)$ (cost function) on the generated pump pressure drop and temperature.

Naturally $P \geq 0$. The optimal pressure is related to the pumping temperature. When solving the problem, instead of the continuous function $S(P, T)$, its discrete analog is used. The pressure value is presented discretely with a rather small step $\varepsilon_p = 0.01$ bar, the temperature value is $\varepsilon_T = 0.05$ °C.

The problem solution is stored in a discrete array $\text{Info}(P, T)$, which for each value of P contains a list of required pumps of several previous stations.

The cost function $S(P, T)$ and the array of solutions $\text{Info}(P, T)$ for the pump have the form:

$$\begin{aligned} S(P, T) &= \begin{cases} +\infty, & P \neq P_p \\ zN^{pu}(Q, T), & P = P_p \end{cases} \\ \text{Info}(P, T) &= \begin{cases} 0, & P \neq P_p \\ (\text{pump number}), & P = P_p \end{cases} \end{aligned} \quad (4.1)$$

where z is the electricity cost at the station (\$/(kW h)), N^{pu} is the pump power (kW).

Q is the flow rate, which flows through the pump.

Pressure drop P_p , created by the pump, is located as:

$$P_p = [\rho g H(Q, T)]$$

where the operator $[\]$ means rounding to the nearest rational number with a step ε_p .

The initial cost function is defined as follows:

$$S_{\text{in}}^{\text{st},1}(P, T) = \begin{cases} 0, & P = P_{\text{init}} \text{ И } T = T_{\text{init}} \\ +\infty, & \text{otherwise} \end{cases} \quad (4.2)$$

We denote by the “union” of two functions S^A and S^B such the function $S(P)$, which has a value for each P :

$$S(P, T) = S^A(P, T) \cup S^B(P, T) = \min(S^A(P, T), S^B(P, T)).$$

Similarly, we denote by the “union” of two arrays Info^A and Info^B such the array $\text{Info}(P, T)$, which has a value for each P :

$$\text{Info}(P, T) = \text{Info}^A(P, T) \cup \text{Info}^B(P, T) =$$

$$= \begin{cases} \text{Info}^A(P, T), & S^A(P, T) \leq S^B(P, T) \\ \text{Info}^B(P, T), & S^A(P, T) > S^B(P, T) \end{cases}$$

We denote by the “overlying” of function S^B to function S^A such the function $S(P, T)$, which has a value for each P :

$$\begin{aligned} S(P, T) &= S^A(P, T) \leftarrow (S^B) = \\ &= \min(S^A(P, T), S^A(P - P^*, T) + S^B(P^*, T)), \end{aligned}$$

where the value of variable P^* for a specific value P determines as:

$$P^* = \arg \min_{P^* \in [0, P]} (S^A(P - P^*, T) + S^B(P^*, T)).$$

Similarly, we denote by “overlying” of the array Info^B to the array Info^A such the array $\text{Info}(P, T)$, which has a value for each P :

$$\begin{aligned} \text{Info}(P, T) &= \text{Info}^A(P, T) \leftarrow \text{Info}^B(P, T) = \\ &= \begin{cases} \text{Info}^A(P, T), & S^A(P, T) \leq S^A(P - P^*, T) + S^B(P^*, T) \\ \text{Info}^A(P - P^*, T) + \text{Info}^B(P, T), & \text{otherwise} \end{cases} \end{aligned}$$

In the mode with heating oil, the cost functions $S_v^{h,i}$ and the solution arrays $\text{Info}_v^{h,i}$ for each i th furnace is considered. The first argument P indicates the pressure drop at the station, the second argument T is the oil temperature at the inlet to the furnace, an additional third argument ΔT is the increase in the temperature of the total flow rate Q when using the furnace.

The cost function for the furnace has an additional index v , the additional index v is the number of sampling of operating furnaces at the station from the set m^h . Naturally, $v \leq 2^{m^h} - 1$.

$$\begin{aligned} S_v^{h,i}(P, T, \Delta T) &= \begin{cases} z^h Q_i^f, Q_{i,v}^f(P, \Delta T) \in [Q_{\min,i}^f, Q_{\max,i}^f] \\ \text{and } T + \Delta T^h \leq T_{\text{oil}}^{\max} \\ +\infty, & \text{otherwise,} \end{cases} \\ Q_{i,v}^f(P, \Delta T) &= \frac{\Delta T^h(P, \Delta T)}{C_i^h Q_i^h(P, g^h(v))}, \\ \Delta T^h(P, \Delta T) &= \frac{Q}{Q_i^h(P, g^h(v))} \Delta T, \\ \text{Info}_v^{h,i}(P, T, \Delta T) &= \begin{cases} (\text{furnace number}) + Q_i^f, Q_{i,v}^f(P, \Delta T) \\ \in [Q_{\min,i}^f, Q_{\max,i}^f] \\ \text{“furnace does not operate”,} & \text{otherwise,} \end{cases} \end{aligned} \quad (4.3)$$

where $Q_{i,v}^f$ is the necessary fuel consumption for the i th furnace during the sampling v , ΔT^h is the flow heating temperature through this furnace.

The vector $g^h(v)$ means the operating combination of heating furnaces during the sampling v . In the obtained solution, the restrictions on the fuel consumption of the furnace (2.6) and the maximum heating in the furnace (3.14) are automatically taken into account, since when determining $S_v^{h,i}(P, T, \Delta T)$ the operation “overlying” is performed on the values of $Q_{i,v}^f$ and ΔT^h .

After calculating all $S_v^{h,i}$ and $\text{Info}_v^{h,i}$, the cost function S_v^{OHS} should be defined and an array of solutions $\text{Info}_v^{\text{OHS}}$ of the total furnace operating at the station for each sampling v . The values S_v^{OHS} and $\text{Info}_v^{\text{OHS}}$ are determined by the “overlying” functions of all operating heating furnaces:

$$\begin{aligned} S_v^{\text{OHS}}(P, T, \Delta T) &= S_v^{h,1} \leftarrow S_v^{h,2} \leftarrow \dots \leftarrow S_v^{h, \text{goper}(v)} \\ \text{Info}_v^{\text{OHS}}(P, T, \Delta T) &= \text{Info}_v^{h,1} \leftarrow \text{Info}_v^{h,2} \leftarrow \dots \leftarrow \text{Info}_v^{h, \text{goper}(v)}, \end{aligned} \quad (4.4)$$

where $\text{goper}(v)$ is the number of operating furnaces during the sampling v .

The operation “overlaying” for the cost functions of furnaces A and B will have the following form (similarly for the array of solutions):

$$S^{h,A}(P, T, \Delta T) \leftarrow (S^{h,B}) \\ = \min_{\Delta T^* > 0} (S^{h,A}(P, T, \Delta T) + S^{h,B}(P, T, \Delta T - \Delta T^*)). \quad (4.5)$$

For any triple of values $(P, T, \Delta T)$, the optimal station operation signifies the optimal selection of operating furnaces from m^h furnaces. Therefore, the optimal cost function for a station is a “union” of all possible cost functions from $(2^{m^h} - 1)$ samples (similarly for the array of solutions):

$$S^{OHS}(P, T, \Delta T) = \bigcup_{v=1}^{2^{m^h}-1} S_v^{OHS}(P, T, \Delta T). \quad (4.6)$$

In the case when none of the furnaces at the station is operating, the calculated cost function and the array of solutions need to be adjusted:

$$S^{OHS}(0, T, 0) = 0, \text{Info}^{OHS}(0, T, 0) = \text{“the furnaces don’t operate”}.$$

Since, when the oil passes through the station (as opposed to a group of pumps), the flow temperature increases and the pressure decreases, the “overlaying” of the heating cost function S^{OHS} on the cost functions of the entire station looks like in the following way (similarly for the array of solutions):

$$S(P, T) = S^{\text{st}}(P, T) \leftarrow S^{OHS} \\ = \min_{P^*, \Delta T^* > 0} (S^{\text{st}}(P + P^*, T - \Delta T^*) + S^{OHS}(P^*, T - \Delta T^*, \Delta T^*)).$$

If the heating furnaces are located after the pumps, then the cost function at the station inlet and the array of its solutions must first be “overlay” by the cost function S^{OHS} , and then by the cost functions of the pump group. Otherwise, the sequence is reversed. If the station does not have heating furnaces, then there is no need to calculate the function S^{OHS} and carry out “overlay” on the cost function of the station.

The pressure drop and the temperature drop at the section depend on the initial oil temperature. To properly “cut” the cost functions and arrays of solutions at the outlet from the station and determine their corresponding values at the inlet of the next station, discrete functions $\Delta P_i^{\text{sec}}(T)$, $\Delta T_i^{\text{sec}}(T)$, $\max_ \Delta P_i^{\text{sec}}(T)$, $\max_ \Delta T_i^{\text{sec}}(T)$ should be calculated for each i th section with a step ε_T .

Moreover, the function $\max_ \Delta T_i^{\text{sec}}(T)$ determines the maximum value of the flow temperature drop between the start point and any point on the i th section.

Thus, the functions $P_{\text{out}}^{\min,i}(T)$, $P_{\text{out}}^{\max,i}(T)$ can be determined for the minimum and maximum allowable pressure at the outlet of the station:

$$P_i^{\text{out}2} \geq P_{\text{out}}^{\min,i} = \max(P_{i+1}^{\text{in min}} + \Delta P_i^{\text{pl}}, \max_ \Delta P_i^{\text{pl}})$$

$$P_i^{\text{out}2} \leq P_{\text{out}}^{\max,i} = \min(P_{i+1}^{\text{out}2 \max}, P_i^{\text{pipe max}})$$

In this case, the condition (14) of the minimum required flow temperature at the outlet from the station will take the form:

$$T_{\text{out}}^{\min,i}(T) = T_{\text{oil}}^{\min} + \max_ \Delta T_i^{\text{sec}}(T).$$

For the problem under consideration, the “cutting” of the cost function at the outlet from the station $S^{\text{sec}}(P, T)$ is redefined by functions $P_{\text{out}}^{\min}(T)$, $P_{\text{out}}^{\max}(T)$, $T_{\text{out}}^{\min}(T)$ and is denoted as the function $S^{\text{st.cut}}(P, T)$.

$S^{\text{st.cut}}(P, T)$ for each P and T has a meaning:

$$S^{\text{st.cut}}(P, T) = \text{CUT}(S^{\text{st}}(P, T), P_{\text{out}}^{\min}, P_{\text{out}}^{\max}, T_{\text{out}}^{\min}) \\ = \begin{cases} +\infty, P \notin [P_{\text{out}}^{\min}(T), P_{\text{out}}^{\max}(T)] \\ +\infty, T < T_{\text{out}}^{\min}(T) \\ S^{\text{st}}(P, T), \text{ otherwise.} \end{cases}$$

The array of solutions is “pruned” in a similar way.

Since the pressure drop at the furnace has already been added to the cost function and the array of solutions at the outlet from the station, then when “pruning” $S_{\text{out}}^{\text{st}}(P, T)$, the changed pressure conditions (9) and (10) are automatically taken into account.

The cost function at the inlet of the subsequent station $S_{\text{in}}^{\text{subs.st}}(P, T)$ is determined by the operation “shift” using the cost function at the outlet from the station $S_{\text{out}}^{\text{st}}(P, T)$ through $\Delta P^{\text{sec}}(T)$ and $\Delta T^{\text{sec}}(T)$. This function has the meaning for each P and T :

$$S_{\text{in}}^{\text{subs.st}}(P, T) = \text{SHIFT}(S_{\text{out}}^{\text{st}}(P, T), \Delta P^{\text{sec}}, \Delta T^{\text{sec}}) \\ = \min_{T^*} (S_{\text{out}}^{\text{st}}(P + \Delta P^{\text{sec}}(T^*), T + \Delta T^{\text{sec}}(T^*))).$$

The operation “shift” for the array of problem solutions of the $\text{Info}(P, T)$ is carried out in a similar way.

Thus, all the operations for finding the optimal mode of “hot” oil pumping have been defined above.

These operations are carried out at all stations, except for the last one, in the order of their location on the pipeline section.

The problem solution is conducted in a cycle shown in the diagram (see Fig. 2) using the initial condition (4.2).

Based on the results of cycle, the optimal final pressure and temperature are calculated:

$$(P^{\text{answ}}, T^{\text{answ}}) = \argmin_{T \geq T^{\text{end}}, P \geq P^{\text{res}}} (S_{\text{in}}^{\text{st}, n+1}(P, T)).$$

where T^{end} is the inlet temperature at the end station, P^{res} is the pressure required to pump oil into the final reservoir.

The minimum sum of consumed energy costs at performance Q is the value of the function $S_{\text{in}}^{\text{st}, n+1}(P^{\text{answ}}, T^{\text{answ}})$. Data on the optimal combination of operating pumps and heating furnaces, as well as modes of their operation and the required pressure drops at heating stations will be stored in the array cell $\text{Info}_{\text{in}}^{\text{st}, n+1}(P^{\text{answ}}, T^{\text{answ}})$.

5. Example of solving

5.1. Calculation of pressure and temperature

We applied the suggested algorithm to the Kassymov–Bolshoy Chagan trunk oil pipeline. It contains several oil pumping stations and heating points. Oil is heated and pumped at these stations and at the heating point Sakharniy. The internal diameter of the Kassymov–Sakharniy pipeline section is $D_1 = 1.0$ m, and the Sakharniy–Bolshoy Chagan pipeline section is $D_1 = 0.7$ m. It can be seen that the diameter of the pipeline is much smaller than its length $L = 450$ km.

At the first step, we just calculated the pressure and temperature distribution of oil along the pipeline for a given flow rate Q and compared with factual data obtained by SCADA. The pressure (top diagram) and temperature distribution (bottom diagram) of oil at the flow rate 1865 m³/h are shown in Fig. 3. The bottom diagram also shows soil temperature distribution. Oil is pumped by pumps at the Kassymov and Inder stations and heated up at the Kassymov, Inder and Sakharniy stations.

The calculated pressure, temperature was compared with the actual data SCADA system (points). Fig. 3 shows the results of calculations and measurements of oil parameters along the pipeline length. The oil pressure drop in the heating furnaces leads to a step change in pressure at the Kassymov, Inder and Sakharniy stations.

The running pumps consume 4244 kW, i.e. 2.79 kW h/ton. The cost of energy consumed by pumps is 202.8 \$/h and 129.39 \$/h for heating furnaces. The specific cost of pumping and heating is 0.133 \$/ton and 0.085 \$/ton, respectively.

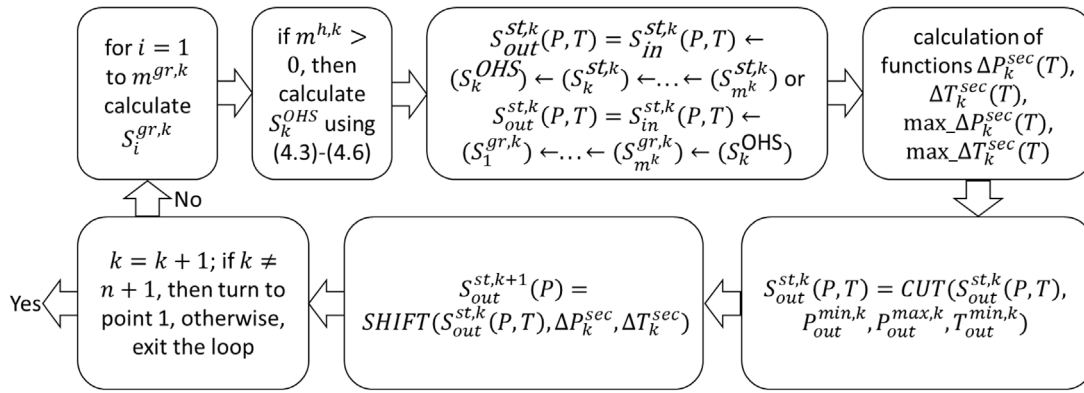


Fig. 2. Diagram of the sequence algorithm for determining the optimal operating mode of pumps and heating furnaces.

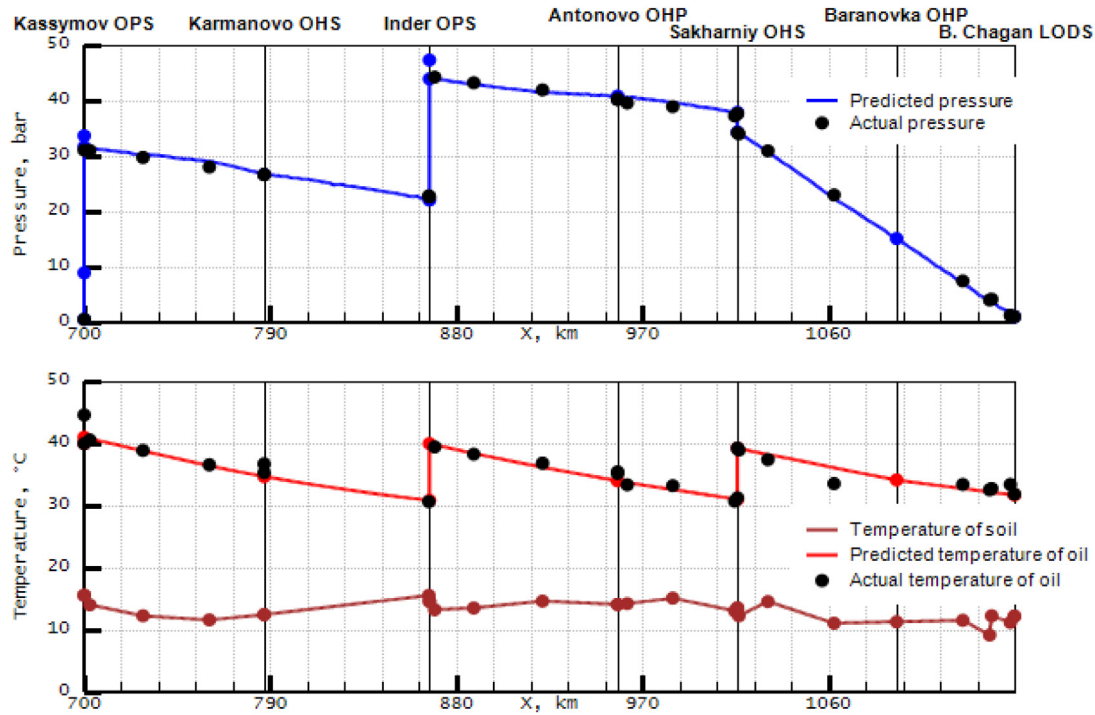


Fig. 3. Distribution of pressure (the blue line on the top diagram) and temperature (the red line on the bottom diagram) of oil along the pipeline length. The flow rate is 1865 m³/h. On the graphs: lines are calculated data, points are the actual data of the SCADA system.

5.2. Optimization of the temperature field

The cost of heating far exceeds the pumping cost, so the optimization of the temperature field in the pipeline is the major objective. As mentioned in the Introduction, the optimal temperature really exists and is nontrivial.

The calculated data for the same oil flow rate of 1148 m³/h are shown in Figs. 4 and 5.

Fig. 4 gives the results of simulation of the pressure and temperature according to the factual data provided by SCADA. They correspond to non optimal regime. As seen, the oil temperature at three stations are 47 °C (Kassymov), 46 °C (Inder) and 50 °C (Sakharney).

Fig. 5 shows the results of simulation in the optimal regime. It was assumed that the temperature does not drop below 28 °C. This is due to the fact that below 28 °C oil can be non-Newtonian, which changes the calculation method.

The optimal temperatures are 40 °C (Kassymov), 35 °C (Inder) and 35 °C (Sakharney).

In the optimal regime, the total power of the pumping units is 3238 kW, while it is 1515 kW in the factual mode. Thus, optimization is achieved by reducing the oil heating temperature.

In the energy saving mode, the total cost of pumping and heating of oil is reduced from 321.1 \$/h to 265.3 \$/h, and unit costs are reduced from 0.326 \$/ton to 0.199 \$/ton.

Thus, the optimal control of the operation of pump units and heating furnaces allows for determine the energy saving of “hot” oil pumping.

Conclusion

1. The minimum value of the sum of the total cost of electricity for pumping units and fuel from heating furnaces is a criterion for optimizing the “hot” pumping of high-viscosity and high pour-point-oil in the section of the main pipeline with several pump and heating stations. The fuel cost of a heating furnace is much higher than the cost of electricity consumed by pumping units, which explains the need to minimize the amount of the cost of pumping and heating, rather than the amount of energy. This explains the need to compare the total cost with the costs that are used to operate pumping units and heating furnaces.

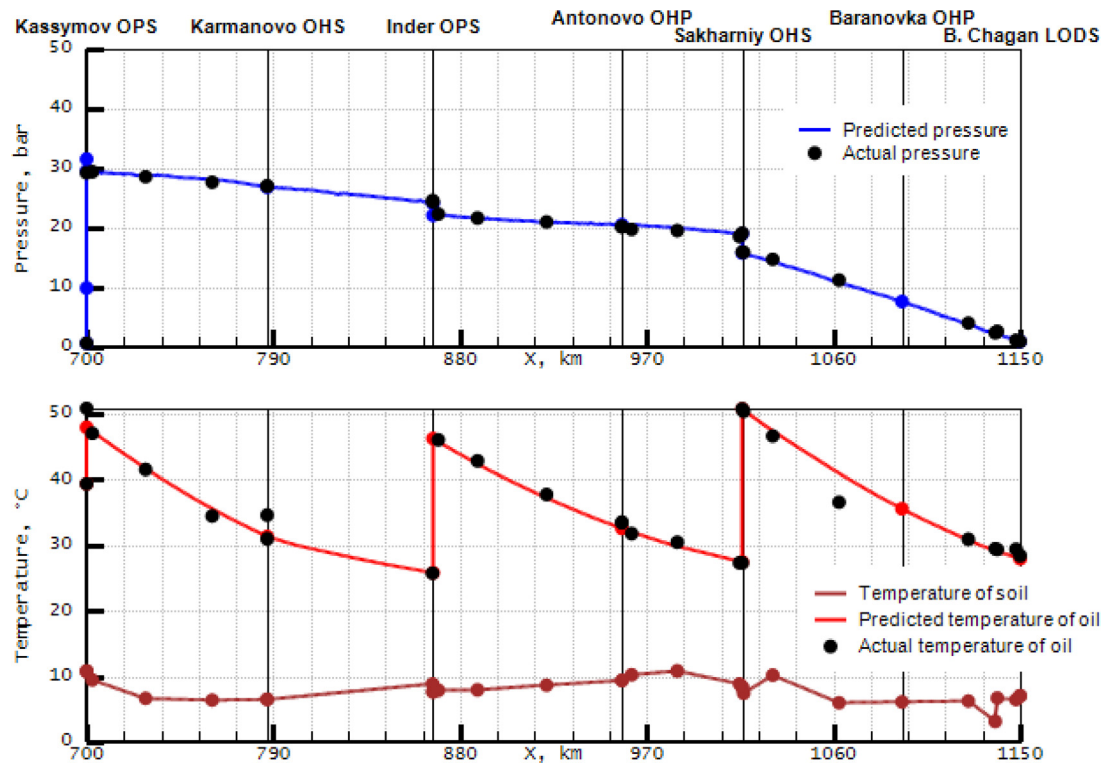


Fig. 4. Distribution of pressure (the blue line on the top diagram) and temperature (the red line on the bottom diagram) of oil according to the SCADA system along the pipeline length. The flow rate is 1148 m³/h. On the graphs: lines are calculated data, points are the actual data of the SCADA system.

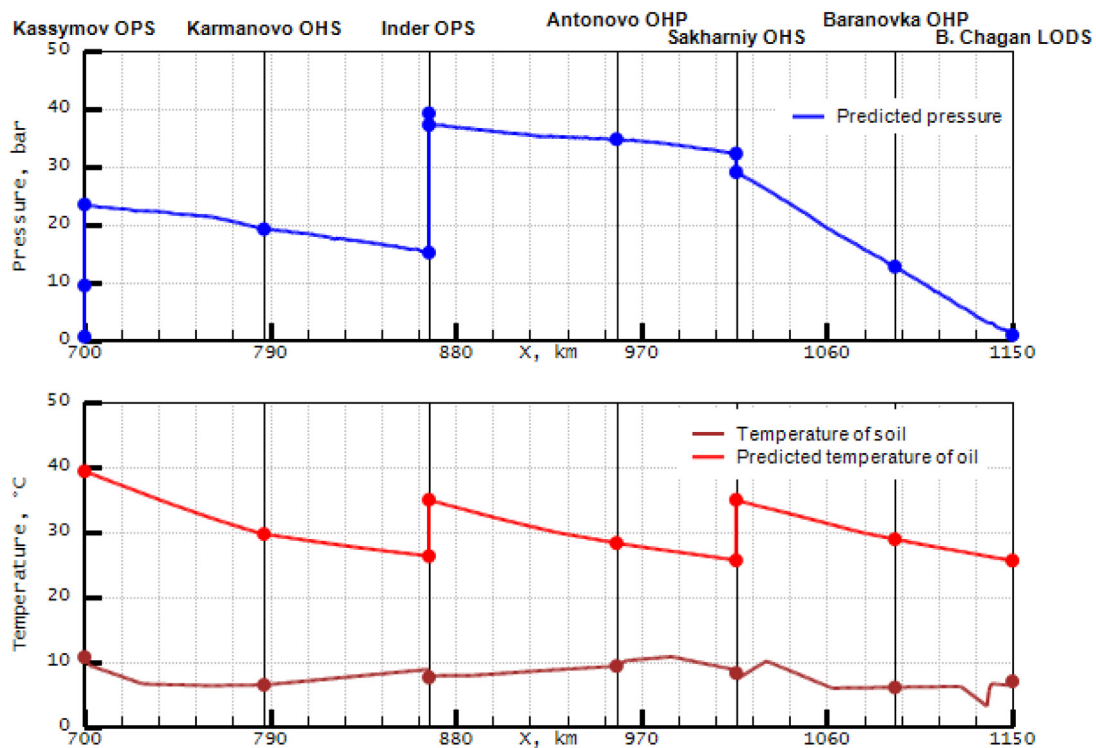


Fig. 5. Distribution of pressure (the blue line on the top diagram) and temperature (the red line on the bottom diagram) of oil along the pipeline length in energy saving mode. The flow rate is 1148 m³/h.

2. We have formulated the optimal problem of “hot” pumping in the form of the criterion (2.1) and several safety restrictions. We developed the general algorithm of solving this problem

(see Fig. 2). It includes as a part the calculation of pressure and temperature field along the pipeline as the numerical solution of the system of differential equations (3.5), (3.6). Note that this

model takes into account the non-stationarity of the temperature field.

3. The SmartTran code implements a general optimization algorithm based on graph theory and dynamic programming, taking into account the safe operation of a heated pipeline.

4. The simulation results shows agreement between that the calculated data obtained by the code SmartTran with the actual data of the SCADA system.

5. For the Kassymov-Bolshoy Chagan pipeline, the optimal operating conditions for pumping and heating units have been determined. Unit costs of energy consumption for pumping and heating are reduced from 0.326 \$/ton to 0.199 \$/ton and the savings are 38.9%.

CRediT authorship contribution statement

Timur Bekibayev: Methodology, Software, Writing - Original Draft. **Uzak Zhabbasbayev:** Supervision, Conceptualization, Writing - review & editing. **Gaukhar Ramazanov:** Funding acquisition, Investigation, Visualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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